

Local disalignment can promote coherent collective motion through rapid information transfer

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Abstract. When particles move at a constant speed and have the tendency to align their directions of motion, ordered large scale movement can emerge despite significant levels of noise. Many variants of this model of self-propelled particles have been studied to explain the coherent motion of groups of birds, fish or microbes. Here, we generalize the exactly aligning collision rule of the classical model of self-propelled particles to the case where particles after the collision tend to move in slightly different directions away from each other, as characterized by a collision angle α . We map out the resulting phase diagram and find that, in sufficiently dense systems, small disalignment can lead to higher global alignment of particle movement directions. We show that in this dense regime, global alignment is accompanied by a grid-like spatial structure which allows information to rapidly percolate accross the system by a “domino” effect. Our results elucidate the relevance of disalignment for the emergence of collective motion in models with exclusively repulsive interaction terms.

1. Introduction

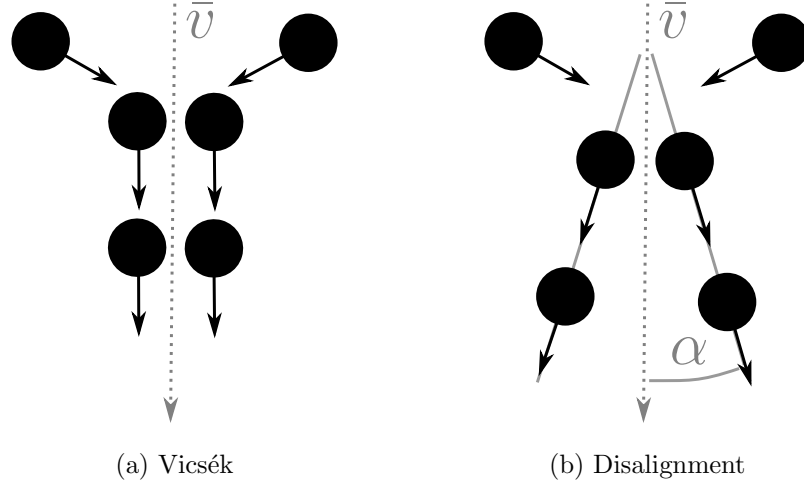


Figure 1: Self-propelled particle models describe the coherent motion of particles that move at a constant velocity and interact when they collide. In the classical model (a) due to Vicsek et al. [1], colliding particles align their direction of motion. The direction after the collision is obtained from averaging the velocities prior to the collision. b) Here, we study a more general model in which the velocity directions after the collision deviate by an angle α from the averaged direction. The case $\alpha = 0$ leads to the original Vicsek model.

The emergence of ordered motion in groups of interacting particles that move at a constant speed is reminiscent of the collective motion observed in many animate and inanimate systems [2]. A wide variety of different models of such self-propelled particles (SPPs) have been explored with the goal to quantify conditions for the global alignment of the movement directions of the individual particles. These models have in common that they rely on local interaction rules, as they are thought to apply to many animal swarms [2, 3, 4, 5, 6, 7, 8], and that the movement directions of particles are continually perturbed by random noise.

The first and most basic of these models of SPPs is due to Vicsek et al. [1], and relies on an explicit alignment interaction that adjusts each particle's movement direction to the average direction of its surrounding particles, see Fig. 1a. For sufficiently weak noise levels, self-organized collective motion results from the local interaction rule that the movement directions of colliding individuals are aligned. For strong noise levels, the system will inevitably fail to order globally. The amount of coherent collective motion can be measured by an 'order parameter' defined as the magnitude of the globally averaged velocity vector. While an analog of an equilibrium phase transition is obtained in the limit of zero velocities, the order-to-disorder transition is generally a unique non-equilibrium phenomenon as it is driven by the perpetual motion of the interacting entities [1]. Whether the transition between the two states is continuous or

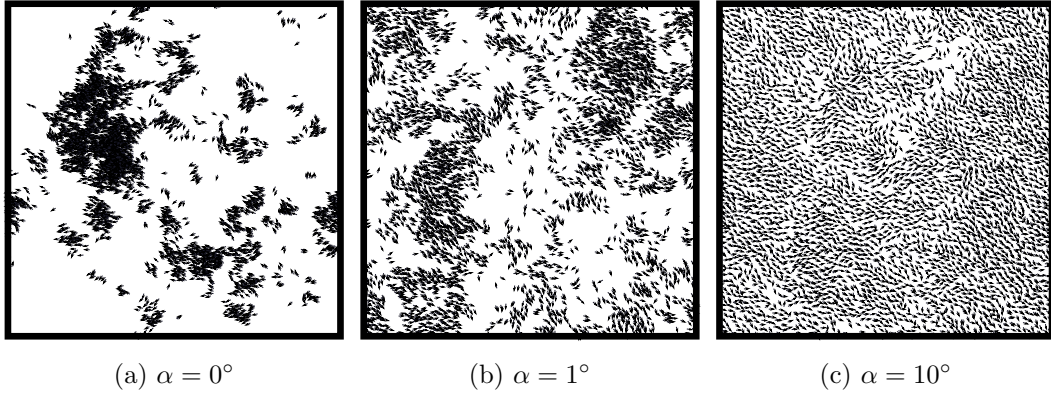


Figure 2: Characteristic snapshots of the distribution of particle positions and movement directions (arrows) in our disalignment model. (a) For vanishing disalignment angle α , the original Vicsék model is obtained with its characteristic flocking structure. As the disalignment angle is increased (b, c), the distribution of particles changes markedly - density fluctuations are suppressed. As we argue in this article, the resulting homogeneous distribution of particles leads to a change in the mechanism that drives global order. (Other simulation parameters were $N = 4096$, $\eta = 25^\circ$)

discontinuous (in the thermodynamic limit) has been intensely debated [2, 9, 10, 11]. Similar phase transitions are observed in variants of the classical Vicsék model that add cohesive and a repulsive interaction terms [2, 7, 12, 6].

Contrary to the Vicsék model and its variants, a second group of SPP models [13] does not introduce an explicit alignment but only an isotropic repulsive force, repelling nearby particles. Surprisingly an ordered phase can be observed even then: The perpetual motion of the SPPs leads to a weak alignment through each collision and, when enough collisions are accumulated, order emerges given weak enough noise [13, 2].

The observation of local alignment causing global alignment, replicated many times, suggests that higher local alignment will always lead to stronger ordering. Furthermore one might think that the Vicsék model, perfectly aligning the SPPs locally, exhibits the highest levels of global order among all models with the same noise strength, particle density and interaction range.

We demonstrate in the following that, contrary to this intuition, local disalignment can even enhance global order. To show this, we generalize the Vicsék collision rule such that the velocity vectors after collision diverge by a small angle α . The disalignment angle α is chosen to point away from the center of mass of the interacting particles, which results in a repulsive interaction, see Fig 1b. For any finite disalignment angle, our interaction rule tends to reduce local order compared to the classical Vicsék model. Yet, our numerically determined phase diagram shows that global order can be increased for a nonzero disalignment, for certain densities and noise levels. We argue that this effect is ultimately the manifestation of reduced density fluctuations in the presenced of disalignment, which leads to a more efficient information transfer across the system.

2. The disalignment model

Our model is a generalization of the classical Vicsek model [1]. In two dimensions, the orientation of each particle can be characterized by an angle Φ , measured in the counter-clockwise direction. In each timestep, the angle Φ_i corresponding to a focal particle i is updated according to the rule

$$\Phi_i(t + \Delta t) = \overline{\Phi}_i^{(r)}(t) \pm \alpha + \Delta\Phi, \quad (1)$$

as illustrated in Fig. 1b. Here, the angle $\overline{\Phi}_i^{(r)}(t)$ characterizes the average orientation of all particles within a circle of radius r centered at the focal particle. The random noise term $\Delta\Phi$ is chosen uniformly from the interval $[-\eta, \eta]$. Both, the level of noise η and the disalignment angle $0 < \alpha < 180^\circ$ are measured in arcdegrees in the following. All angles are measured in the counter-clockwise direction. Finally, the parameter $\alpha \geq 0$ is the disalignment angle – the new parameter of our model. The sign in front of α is always chosen such that the particles tends to move away from the line that projects from the average position along the average movement direction of all particles inside the interaction radius \ddagger . As a consequence, the α term drives particles away from one another, as illustrated in Fig. 1a. Note that the original Vicsek model is obtained when the disalignment angle α is set to zero. The parameter α can also be viewed as a tuning wheel by which the exact alignment interaction can be broken systematically.

The position \vec{x}_i of particle i is consequently updated as:

$$\vec{x}_i(t + \Delta t) = \vec{x}_i(t) + v \begin{pmatrix} \cos(\Phi_i(t + \Delta t)) \\ \sin(\Phi_i(t + \Delta t)) \end{pmatrix} \Delta t \quad (2)$$

Throughout the reported simulation results, we chose the magnitude of the particle velocity constant as $v = 0.1$, the timestep $\Delta t = 1$ and the interaction radius $r = 1.0$. The N particles move in a square cell with periodic boundaries of length L . The particle density is given by $\rho = N/L^2$.

We measure the degree of global alignment by the order parameter φ ,

$$\varphi = \frac{1}{|v|} \left| \sum_i^N \vec{v}_i \right|, \quad (3)$$

which represents the normalized average particle velocity.

3. Snapshots and phase diagrams

After randomizing the initial particle positions and orientations, the particles start to move and interact through collisions. For small disalignment α , each collision tends to align the colliding particles and, as in the original Vicsek model, dense groups of aligned particles form moving jointly through the system [1], see Fig. 2a. The abundance of particles within an interaction range inside a dense group reduces the effect of noise by averaging the movement directions of many particles. Depending on the noise strength

\ddagger For definiteness, we set $\alpha = 0$ when a particle is alone within its interaction range.

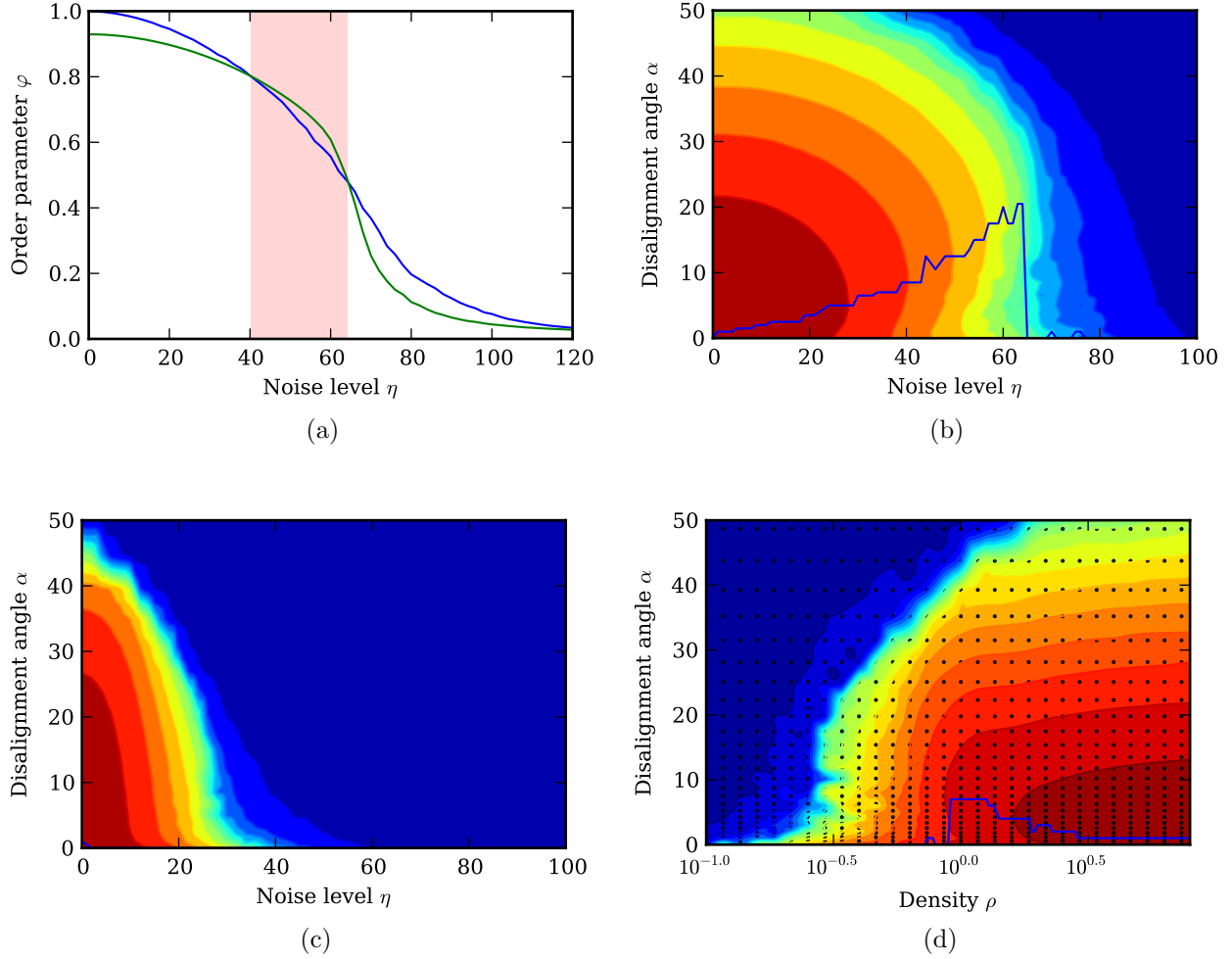


Figure 3: Phase diagrams summarize the global orientational order in our disalignment model, and show that collective motion can be promoted by a small degree of disalignment. a) Order parameter φ as a function of noise η in the classical Vicsék model ($\alpha = 0^\circ$, blue) and the disalignment model ($\alpha = 20^\circ$, green), respectively, at a particle density of $\rho = 2$. For medium noise levels (same density), the disalignment model has a higher order parameter and exhibits a sharper transition to the disordered phase (shaded region). In the heat plots b), c) and d), the order parameter φ is indicated by color (red=1, blue=0) as a function of disalignment angle α and noise η (b,c) and as a function of α and ρ (d), respectively. Figs. b,c) differ in their densities (b: $\rho = 2$, c: $\rho = 0.5$). In Fig. d), the noise level was fixed at $\eta = 20^\circ$. The optimal disalignment angle (maximum in vertical direction) is indicated by the blue line. Measurement points are shown in d) as black dots. Other parameters were $N = 2048$ and $v = 0.1$.

and density, these groups can further align among each other, thus leading to a nonzero order parameter φ . The smaller the number and size of the groups, the less frequent are the interactions between them.

As one increases α to 1° , the formation of dense groups is suppressed due to the repulsive effect of the disalignment interaction. More loosely connected groups form instead, which occupy a much larger area of our simulation box. On the one hand, this leads to more frequent collisions between clusters, as illustrated by the snapshot in Fig. 2b. On the other hand, the number of particles within an interaction range decreases inside clusters, thus amplifying the effect of noise.

If we further increase α to 10° , dense groups become very rare and a grid-like structure forms that spans most of the simulation area (Fig. 2c). Such a system spanning grid can, however, only form for large enough mean densities, $\rho \gtrsim 1$. For very low densities, the repulsive interaction disintegrates any cluster of particles such that solitary particles move in random directions.

The phase diagrams in Fig. 3 summarize the behavior of our model. The first plot (Fig. 3a) shows the behavior of the order parameter φ as a function of noise level η for the original Vicsék model with $\alpha = 0$ and for finite disalignment with angle $\alpha = 20^\circ$. Both systems are highly ordered for small noise and become disordered as noise levels are increased. At zero noise, the Vicsék model approaches perfect global order with $\varphi = 1$ while the disalignment model retains less order, as one might expect. At medium noise levels, however, the disalignment rule for $\alpha = 20^\circ$ leads to higher global order than for $\alpha = 0^\circ$. This counterintuitive behavior at intermediate noise levels is the focus of our discussion below. Also note that the transition from the ordered to the disordered phase appears to be sharper for disalignment than for alignment.

The heat plot in Fig. 3b shows a two-dimensional phase diagram, in which the order parameter is indicated as a function of both the noise level and the disalignment angle, for a similar density as in Fig. 3a. Again, the asymptotic behavior follows intuition: high noise levels and large disalignment angles together prevent order. Highest order is achieved for zero noise levels and zero disalignment angles. The intermediate behavior, however, again shows the surprising phenomenon of an 'optimal' disalignment angle that leads to the highest order for a given noise level η . This angle is indicated as a blue line and increases with increasing noise levels.

Disalignment can only promote global alignment when densities are of order ≈ 1 or larger. For lower densities, the order parameter is always largest for vanishing disalignment, as can be appreciated from Fig. 3c $\rho = 0.5$. At these densities, smaller noise levels suffice to break down order, even more so for finite disalignment angles.

The heat plot in Fig. 3d finally depicts the dependence of the order on both the disalignment angle and the particle density, for a fixed level of noise. As one increases the density of the system, an optimal disalignment angle appears at $\rho \approx 1$, and decreases as one further increases the density of the system. The sharp contrast of the optimal angle at ρ greater or less than ≈ 1 and the different behavior indicated in the snapshots in Fig. 2a suggests that the spatial distribution of the particles, in dense groups or a grid-like structure, could play a crucial role for explaining our results.

4. Information transfer

4.1. The role of information transfer

In models of collective motion, global coherent order emerges from an interplay between aligning and random disaligning forces. The aligning forces allow orientational information to be transmitted from one particle to its close neighbours. The global effect of these driving forces on order does not only depend upon the degree of alignment in an interaction. It also depends to a large extent on the ability of the system to exchange information about movement directions between all particles: A particle that never interacts with a group of aligned particles will never align with them, no matter how strong the aligning force might be. On this view, low density therefore should generally decrease the order of a system because the number of interactions between particles is reduced such that information about the orientation of a particle travels more slowly through the system while noise deteriorates the information during the transmission. This behavior has often been observed in previous studies [1, 14, 15] and also characterizes our model, see Fig. 3d.

In models of collection motion, information about movement directions spreads in two distinct ways:

i) Neighboring particles closer than the interaction distance directly adjust their movement directions with respect to one another. The orientation of particles within a connected clusters can be aligned after a few timesteps by this direct interaction through a 'domino' effect: First neighbours within an interaction range align and encode the information about the directions of their neighbors in their own new direction. In the next step, they propagate this information to their nearest neighbours such that information spreads until there are no more new particles within an interaction range of the considered cluster of particles. While the positions of all particles change during this process, the information transmission does not rely on particle movement (in contrast to the second mechanism of information transmission, below) - it is also present in the limit of zero velocities.

ii) The second type of information flow in models of collective motion depends on the movement of particles. This property fundamentally distinguishes self-propelled particle models from equilibrium models. A particle can move to a distant location and exchange orientational information with a particle there. Or in the context of the whole system, a particle can be used as a 'messenger' that exchanges orientational information between two or more other particles. In contrast to the above described direct interaction, this mechanism works over large distances even without a continuous chain of neighbours that are separated by at most one interaction distance. However, since information deteriorates by noise while a particle is on its way, single particles can transmit information only over short distances. Therefore, dense groups of interacting particles, or "flocks", play an important role for preserving orientational information against randomization through noise: A group continually averages the orientations of all of its members and can thus retain its average velocity over longer times and travel

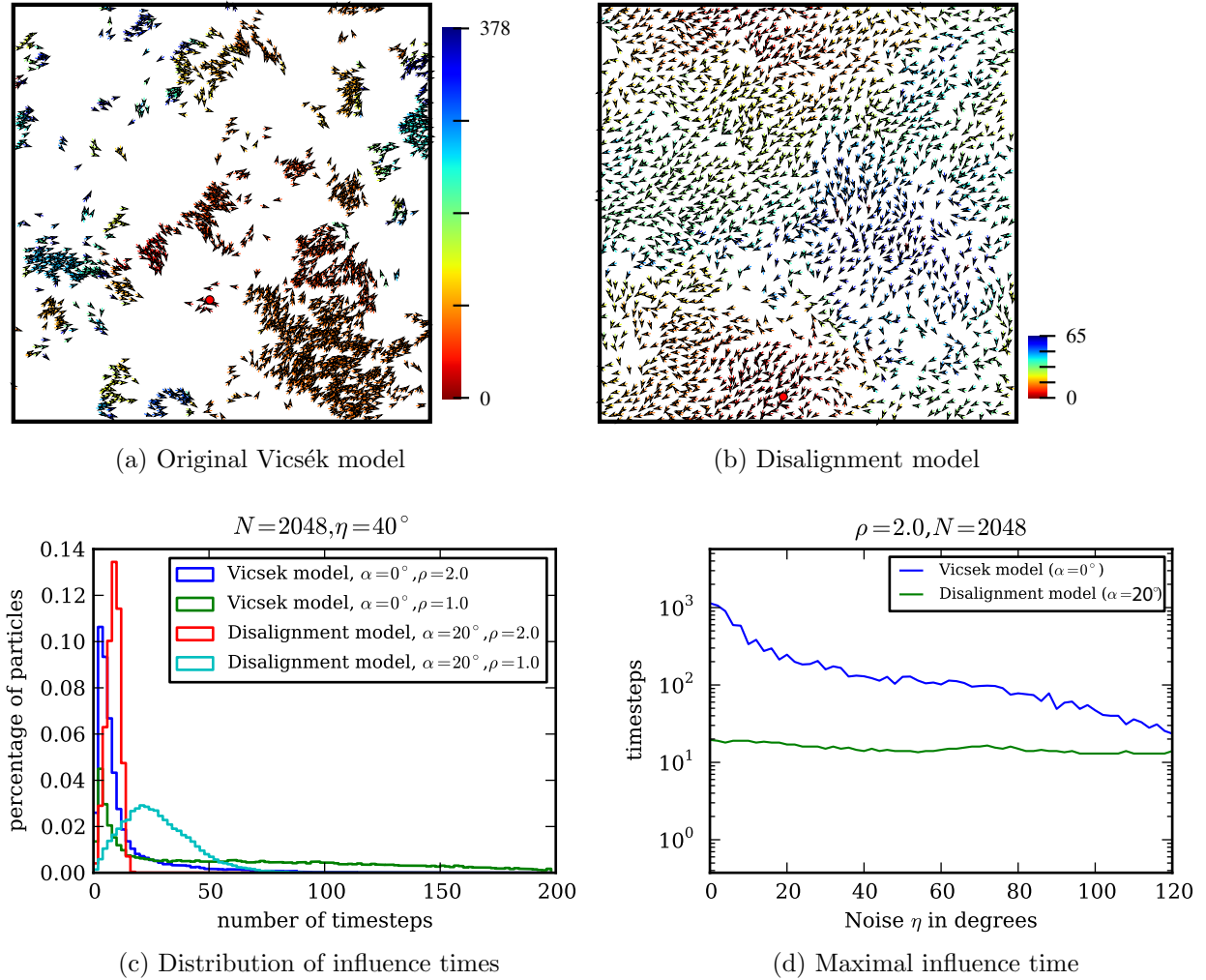


Figure 4: Information spreads differently with and without disalignment. Figs. a, b) show snapshots of particle positions and movement directions (arrows) at a given point in time. The color of a particle codes for the additional delay time until it is “influenced” (as defined in the main text) by the current state of a focal particle (red dot). In Fig. a), original Vicsek model ($\rho = 1.0$, $N = 2048$, $\eta = 30^\circ$), influence spreads in chunks from group to group. In b), disalignment model ($\rho = 1.0$, $N = 2048$, $\eta = 30^\circ$, $\alpha = 10^\circ$), influence spreads evenly and much faster through the system, which is spatially organized in a grid structure. Fig. c) shows distributions of influence times for the Vicsek and the disalignment model ($\alpha = 20^\circ$) at two different densities and equal noise ($N = 2048$, $\eta = 40^\circ$). For both densities, the distribution corresponding to the Vicsek model exhibits a long time tail and a peak, which is due to interactions within and between “flocks”, as we argue in the main text. These features are absent in the grid-structured regime of the disalignment model, where information spreads through a different mechanism. Fig. d) shows the maximal influence times for different noise levels and many realizations. Notice that disalignment (with $\alpha = 20^\circ$) can reduce these maximal influence times by orders of magnitudes.

distances. A clustered structure as in the original Vicsek model is therefore crucial for this type of long distance information transfer [9, 11, 18, 3]).

Information transmission through the directed motion of self-propelled particles is suppressed in the presence of a disalignment rule because the formation of groups is hampered by the repulsive character of the interaction. As a consequence, order breaks down below a certain density when there are not enough neighbours to ensure a continuous chain of information transfer.

For large enough densities, however, a grid-like structure observed at density $\rho > 1$ in the disalignment case allows the whole system to receive orientational information through this mechanism. Information then percolates through the system in a similar manner as in the equilibrium “XY” spin model. For the XY-model, however, it can be theoretically shown that full order can not be achieved in 2 dimensions such that an SPP model certainly requires movement of particles to achieve an ordered state far from equilibrium [16, 17].

For large enough densities ($\rho > 1$), however, a grid-like structure forms in the disalignment case, which allows the whole system to receive orientational information through the first “domino” mechanism. Information then percolates through the system in a similar manner as in the equilibrium “XY” spin model. For the XY-model, however, it can be theoretically shown that full order can not be achieved in 2 dimensions such that an SPP model certainly requires movement of particles to achieve an ordered state far from equilibrium [16, 17].

We expect (and test below) that this domino mechanism of information transfer is typically much faster than information transfer by moving flocks, simply because the signal propagation is independent of the motion of the signal carriers.

4.2. The maximum speed of information transfer

We now demonstrate that information can indeed be transmitted by two different mechanisms at very different speeds depending on the chosen model parameters. To this end, we measured how quickly a particle interacts with all other particles through a chain of successive interactions. This can be done, by choosing a focal particle and following through time how other particles became influenced by this particle. Operationally, we define the set of ‘influenced’ particles as follows: i) At time t_0 no particles is influenced by the focal particle, yet. ii) At $t > t_0$, particles become influenced by the focal particle if they either interact directly with the focal particle, or indirectly via an already influenced particle. The number of influenced particles increases over time and measures an upper limit for the speed at which the focal particle is able to transmit any information to other particles. In Fig. 4 we display the time until particles are influenced by a randomly chosen focal particle, marked by a red dot, for a) the Vicsek model and b) the Disalignment model with $\alpha = 10^\circ$ in a single realization. Particle positions are shown at time t_0 . Notice that the influence of the focal particle spreads much more quickly in the presence of disalignment. Furthermore, information spreads in chunks from group

to group in the Vicsék model. The disalignment model, on the other hand, circulates orientational information of the focal particle evenly throughout the system and reaches the last particles much faster. Note our measure of the speed of information transfer does not entail any statement about the quality of the transmitted information, which deteriorates through the continual perturbation by noise. Nevertheless, it demonstrates the qualitative difference between the two types of information transfer that we described above.

Fig. 4c) shows the measured distributions of transmission times obtained from many runs by averaging over the choice of the focal and influenced particle. Note that the difference between the case with disalignment and with strict alignment are particularly prominent at $\rho \approx 1$, where the Vicsék model exhibits a broad class of particles that need many time steps to receive any information from a focal particle. The distribution is peaked at the characteristic size of particle groups, which exchange information on a fast time scale. In Fig. 4d, we report the time until the last particle of the system is influenced by a focal particle for many realizations. This maximum transmission time is, for low noise, orders of magnitude larger than in the disalignment model. Interestingly, neither of the models displays a sudden change of the maximum influence time at the critical noise level where global order breaks down.

5. Conclusions

We have demonstrated that changing the strictly aligning interaction of the classical Vicsék model to a slightly disaligning interaction can lead to an increase in global order. The beneficial effect of the disalignment term on global order is most prominent for densities close to ~ 1 when the spatial structure of the population was found to be fundamentally changed by the repulsive disalignment interaction. Isolated groups or flocks, as found in the classical Vicsék model, were instabil and disintegrated in the presence of the disaligning interaction. As a consequence, a homgeneous grid-like spatial structure formed that spaned the whole system. Within this structure, orientational information spread locally and rapidly from site to site. This is in contrast to the Vicsék model where information spreads over large distances through the motion of dense groups of particles (“flocks”). These results shows that even a small repulsive term can lead to fundamentally different behavior of the system. Both, the aligning as well as the disaligning part of the interaction can play an important role for the large-scale coherent motion of the particles. There is an abundance of systems that are modeled with repulsive terms in nature [19, 8, 4, 6] (e.g. many particles avoid collisions), in which this effect could be relevant.

Since information transfer in the grid-like and the flocking regimes are fundamentally different, it is not evident that the phase transition from order to disorder is of the same type in both regimes. For instance, it is not clear, whether the phase transition is continuous or discontinuous, or whether one has true long range order in the grid-like regime: Since orientational information spreads in a similar manner than in the

XY-model, noise might destroy long-range order as predicted for 2d equilibrium systems through the Mermin-Wagner theorem [16, 17]. Answering these questions would be the virtue of more extensive simulations than were possible for this first study. Moreover, an analytical treatment of this effect in terms of a kinetic theory [20, 21, 22] as well as a detailed scaling analysis [14] are could help to clarify the key control parameters in our system.

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